properties of a binary mixture with the exception of those for phase equilibrium. It was also verified with the data taken at elevated pressures. Accordingly, it appears to be of practical significance for practising engineer.

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A simple method of dimensioning straight fins for nucleate pool boiling

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INTRODUCTION

MANY METHODS have been proposed to enhance nucleate pool boiling; methods related to surface structures seem most promising. For a detailed literature survey on the subject, the reader is directed to [1,2]. Only the data in the literature relevant to the present study will be briefly mentioned in this paper.

Fundamental work has shown that, for the enhancement of nucleate pool boiling, the surface of a tube should, in general, have either interconnected re-entrant grooves or a porous coating [1]. At present, both the dimensioning of the structure of an enhanced surface and the evaluation of its performance are carried out by experiment, since no predictive correlating equations of broad generality exist in the literature [1, 2].

Four types of commercially available and patented surfaces applied to circular tubes are mentioned. These are the High-Flux, the Thermoexcel-E, the Gewa-T and the ECR40 surfaces.

The High-Flux surface consists of a porous, metallic matrix which is bonded to a metallic substrate. The surface layer is about 0.25–0.50 mm thick with a void fraction of 50–65% and contains a multiplicity of cavities or pores which function as sites for the generation of vapour bubbles [3]. The Thermoexcel-E surface has continuous tunnels and isolated pores on top of the tunnels. The construction is manufactured by bending the ridges of rugged microfins. Typical values for the geometry of this surface are : tunnel width, 0.2 mm; depth, 0.4 mm; pitch, 0.55 mm; pore diameter, 0.1 mm; and number density of pores, 260 cm⁻² [2]. The Gewa-T surface is obtained by flattening the fin tips of a integral fin tube [1, 2]. It is in fact identical to the ECR40 surface, excepting that the flat crests of the fins in the latter are provided with porous plates [1, 2].

Yilmaz and Westwater [4] have measured the performances of copper circular tubes with these four surfaces and also the performance of a plain copper tube during saturated nucleate pool boiling of *p*-xylene and isopropyl alcohol at atmospheric pressure. In accordance with the data obtained by these investigators, the wall superheat required for boiling on these tubes is much lower than that on a plain tube. For a given heat flux and at low wall superheats, the heat transfer coefficients for these tubes are significantly higher than the plain-tube heat transfer coefficient. At high wall superheats however, the improvements in these heat transfer coefficients of these heat transfer coefficients to the plain tube heat transfer coefficient vary between about 1.7 and 6.9 for a heat flux of 60 kW m⁻², and between about 1.3 and 3.3 for a heat flux of 340 kW m⁻². The wall superheat for the plain tube is about 9.8 and

NOMENCLATURE			
A	cross-sectional area of a fin [m ²]	n	a dimensionless constant
$a_1,, a_4$	constants	р	fin efficiency
Ċ	circumference of a fin [m]	Q	rate of heat flow through entire fin [W]
d	outer tube or fin diameter [m]	r	radial coordinate [m]
е	ratio of the rate of heat flow from a finned	ΔT	wall superheat, i.e. the difference between
	tube to that from a plain tube of the same		surface and saturation temperature [K]
	length and O.D.	t	distance between two successive fins on
$F(j/45^{\circ})$	Legendre's normal elliptic integral of the		the outer surface of a tube [m]
	first kind for the modular angle of 45°	и	half fin thickness [m]
f	fin effectiveness	w	fin width [m]
h	heat transfer coefficient $[W m^{-2} K^{-1}]$	x, y, z	Cartesian coordinates [m].
j	amplitude [degrees]		
k	thermal conductivity $[W m^{-1} K^{-1}]$	Subscripts	
L	fin length [m]	ь	fin base
m	number of fins	e	fin tip.

20 K, respectively [4]. Similar results were obtained from the experiments carried out with saturated nucleate pool boiling of R11 and R113 on the sintered layers of copper and bronze particles at atmospheric pressure [5]. Beyond a given superheat, the surfaces with the sintered layers of bronze particles appear to foil the enhancement of heat transfer [5].

Tubes with enhanced nucleate pool boiling surfaces are quite expensive due to the special methods used to manufacture them [6]. They have been mostly applied to halocarbon refrigerants [3, 6, 7]. For a particular application and in comparison with a conventional evaporator, the gains made in the overall heat transfer coefficient of an evaporator with enhanced heat transfer tubes and the thermal efficiency of the system in which this evaporator is placed should be weighed against the losses accruing from the cost of these tubes and a lower logarithmic mean temperature difference in the evaporator.

As indicated by Shah (as discussed in ref. [6]), the operation of a pool boiling enhancement mechanism on a structured surface may fail with a liquid containing oil if this oil is not dissolved in the liquid (ammonia, for example). Hence it is likely that oil will be deposited in the cavities which would then cease to function as bubble-nucleation centers. This should be verified by experiment.

As compared with the tubes with surface structures (High-Flux, etc.), a tube with longitudinally arranged, straight fins of rectangular profile can be produced cheaply by extrusion, and the rate of heat flow from such a tube would appear to be quite significant when compared with that from a plain tube of the same O.D. and length. The object of this study is to present a method for dimensioning such a finned tube.

Firstly, the mathematical background is explained and thereafter the method. Numerical examples illustrate the use of the method which also applies to a pin fin protruding from a heat source.

MATHEMATICAL BACKGROUND

A thin-walled, horizontally or vertically located tube with logitudinally arranged, straight fins of rectangular profile is considered here. Such a tube is shown schematically in Fig. 1a; it is manufactured from a material of high thermal conductivity. For many practical applications of this tube it seems justifiable to assume that it is subjected to a uniform temperature along the whole of its outer circumference including the fin bases at a given axial location z. For example, at a typical design heat flux of 50 kW m⁻² for flooded chillers consisting of copper tubes, the radial temperature drop in the wall of one of the tubes is only about a few tenths of a degree [6, 7].

The fins on the tube are geometrically identical in form, i.e. the fin thickness 2u and fin length L are constant. The distance



a) tube with straight fins of rectangular profile



b) a straight fin of rectangular profile



c) a pin fin

FIG. 1. Geometries considered.

between two successive fins on the outer surface of the tube is also constant. The liquid surrounding the tube is at the state of saturation. Assuming that all the fins on the tube are operating under same conditions, one single fin (shown in detail in Fig. 1b) is analysed, neglecting the heat conduction in the z-direction. If (hu/k), the Biot number for the fin, is less than 0.1, then the effect on the rate of heat flow from the fin of heat conduction in the y-direction appears to be quite negligible, i.e. less than 1% [8]. For the analysis of the fin, the following assumptions are made:one-dimensional steady-state heat conduction through the fin; a constant thermal conductivity for fin material; no heat sources in the fin itself; negligible heat transfer from the fin tip; and a constant fin thickness. With the exception of the last mentioned assumption, identical assumptions were used in most analytical fin studies presented in the literature [9, 10]. In these studies, a uniform heat transfer coefficient was assumed, whilst in this work the coefficient is taken as being nonuniform.

For the conditions now being considered, the differential equation of the temperature distribution in the fin becomes [9, 10]:

$$\frac{\mathrm{d}^2(\Delta T)}{\mathrm{d}x^2} = \frac{h}{k\mu}\Delta T.$$
 (1)

The boundary conditions are:

$$\Delta T = \Delta T_{\rm b} \quad \text{for} \quad x = L \tag{2}$$

$$\frac{d(\Delta T)}{dx} = 0 \quad \text{for} \quad x = 0 \tag{3}$$

h in equation (1), the saturated nucleate boiling heat transfer coefficient, is given by

$$h = a_1 \Delta T^n \tag{4}$$

where a_1 and n are constants for a given boiling surface, liquid and pressure.

Equation (1) can be analytically solved for only a few values of n as discussed in ref. [11]. The most common value proposed in the literature for n in equation (4) is equal to 2 for water, various refrigerants and hydrocarbons [12, 13]. For this particular value of n, the solution of equation (1) is quoted from

ref. [11]:

$$F(j/45^{\circ}) = \Delta T_e a_2 x \tag{5}$$

$$\cos j = \Delta T_{\rm e} / \Delta T \tag{6}$$

where $F(j/45^\circ)$ is Legendre's (incomplete) normal elliptic integral of the first kind for the modular angle of 45°, and :

$$a_2 = (a_1/ku)^{0.5} \tag{7}$$

 $F(j/45^\circ)$ is known for a given value of j as tabulated in ref. [14] and is also given in Fig. 2.

 ΔT_{e} in equations (5) and (6) is determined by solving simultaneously the following two equations, which are obtained by applying the first boundary condition to equations (5) and (6)

$$F(j_{\rm b}/45^\circ) = a_2 L \Delta T_{\rm e} \tag{8}$$

$$\cos j_{\rm b} = \Delta T_{\rm e} / \Delta T_{\rm b}. \tag{9}$$

For a given geometry, operating conditions and ΔT_{b} , i.e. the boundary value—see equation (2), a value for ΔT_{e} is assumed. j_{b} is calculated from equation (9), $F(j_{b}/45^{\circ})$ from equation (8) and the tabulated values of $F(j/45^{\circ})$ (or from Fig. 2). The value of ΔT_{e} is iterated till the calculated two $F(j_{b}/45^{\circ})$ values are equal.

For a given geometry, operating conditions, ΔT_b , ΔT_e and x, ΔT is predicted using the following procedure : firstly, $F(j/45^\circ)$ is calculated with equation (5). The value of j corresponding to this $F(j/45^\circ)$ is then determined by iteration from the tabulated values of $F(j/45^\circ)$ (or from Fig. 2). Using this value of j, ΔT is solved from equation (6).



FIG. 2. Legendre's normal elliptic integral of first kind for the modular angle of 45° [14].

CRITERIA FOR ENHANCED SURFACES

Two criteria are widely used in the literature to characterize a fin, the fin effectiveness and the fin efficiency [9]. For the purpose of this study, the fin effectiveness seems a more proper criterion than the fin efficiency since it is dealt with in the performance of a tube with and without fins. It is the ratio of the heat transferred through the base of the fin to that which would be transferred through the same base area if the fin were not there, the base temperature remaining constant [9].

The fin effectiveness applicable to all values of n, with the exception of n = -2, has been derived for a straight fin of rectangular profile in ref. [11]. For n = 2, it becomes

$$f = a_3 a_4 / \Delta T_e \tag{10}$$

where

$$a_3 = (k/2ua_1)^{0.5} \tag{11}$$

$$a_4 = (\Delta T_e / \Delta T_b)^3 [(\Delta T_e / \Delta T_e)^4 - 1]^{0.5}.$$
 (12)

The total rate of heat flow through the fin follows from the definition of fin effectiveness, i.e.

$$Q = Aa_1 f \Delta T_b^3 \tag{13}$$

where A is the cross-sectional area of the fin perpendicular to x-axis (i.e. A = 2uw).

The relationship between the fin effectiveness and the fin efficiency is [11]

$$f = pL/u. \tag{14}$$

METHOD OF DIMENSIONING A TUBE WITH FINS

Consider now the tube shown in Fig. 1a. For practical applications, k, the thermal conductivity of the fin material; $\Delta T_{\rm b}$, the wall superheat on the outer surface of the tube and on the fin bases; d, the outer tube diameter, and the operating conditions for the liquid surrounding the tube are known. The following values should be determined: L and 2u, the length and thickness of each fin on the tube; t, the distance between two successive fins on the outer surface of the tube; m, the number of the fins on the tube and e, the ratio of the rate of heat flow from the tube to that from a plain tube of the same outer diameter and length. For this purpose, L, u and t are taken as parametric variables, and m and e are calculated as a function of the known and parametric variables. Neglecting the curvature under the fins (i.e. 2 $u/d \ll 1$) and considering equation (4) and the definition of fin effectiveness, the derivation of m and e are straightforward:

$$m = \pi d/(2u+t) \tag{15}$$

$$e = \frac{(2umf + \pi d - 2mu)a_1 \Delta T_b^3 w}{\pi d a_1 \Delta T_b^3 w} = \frac{2um(f-1)}{\pi d} + 1 \quad (16)$$

provided that saturated nucleate pool boiling takes place both on the surfaces of the fins and on the outer surface of the tube between the fins. e is in fact the ratio of heat transfer coefficient of the tube to that of a plain tube of the same O.D. and length if both heat transfer coefficients are based on the plain tube outer surface area and the wall superheat on this area.

During the experiments carried out by Hahne and Müller [15] at atmospheric pressure using R11 and a copper tube with circular fins of rectangular profile (i.e. d = 15.9 mm, L = 1.5 mm; u = 0.2 mm and t = 0.95 mm), it was visually observed that all the spaces between fins were filled with vapour for a heat flux greater than 20 kW m⁻². At heat fluxes lower than this, a number of fin interspaces remained without vapour production. Due to lack of data at the present time and the complexity of the boiling phenomenon, the magnitude of t,

beyond which nucleate boiling takes place on the interfin spaces, appears to be determined by experiment.

In order to calculate e for a given set of operating conditions (i.e. a_1 , k and ΔT_b), and geometry (i.e. d and parametric variables L, u and t), ΔT_e is first solved from equations (8) and (9), as explained previously. For this purpose, Fig. 2 is used in which the range of $\Delta T_e/\Delta T_b$ is divided into two intervals for the sake of accuracy. Thereafter, f is predicted with equation (10) and e with equation (16). It is now a straightforward matter to determine the total rate of heat flow from the tube with the latter equation, i.e. it is equal to the nominator of the first expression in equation (16).

NUMERICAL EXAMPLES

In order to give one an idea of the magnitude of heat transfer enhancement obtained with a tube with straight fins of rectangular profile, some illustrative examples are presented below.

Consider two finned tubes of 22 mm O.D., as shown in Fig. 1a. One of the tubes is manufactured from copper and the other from aluminum. The working medium is saturated R113 at atmospheric pressure. For this pressure and R113, a_1 in equation (4) is equal to 12.1 W m⁻² K⁻³ (as predicted from the correlation given in ref. [13]). This correlation corresponds reasonably well to the data of Nishikawa *et al.* [5]. ΔT_b is taken equal to 13 K, which is lower than the wall superheat corresponding to dryout/burnout heat flux [5, 16]. The parametric variables selected are:

$$L = 2.5, 5$$
 and 7.5 mm
 $u = 0.1, 0.2$ and 0.3 mm
 $t = 1, 2$ and 3 mm.

The results obtained, using the method described previously, are illustrated in Fig. 3 in which e, the ratio of the rate of heat flow from each one of two tubes being considered to that from a plain tube of the same O.D. and length, is given as a function of parametric variables L, u and t. ΔT_e , the wall superheat at the fin tip (not shown in the figure), varies between 7.6 and 12.4 K for the cases being analysed. Since a wall superheat of 7.6 K is higher than that required to initiate boiling for R113 at atmospheric pressure [5, 16], saturated nucleate pool boiling takes place on both tubes. The Biot number for any fin on the tubes is much smaller than 0.1, which justifies the one-dimensional heat conduction assumption [8]. The calculated e applies to a given axial location along the each of the tubes where ΔT_b is equal to 13 K.

It follows from Fig. 3 then that in the presence of saturated nucleate pool boiling, the rate of heat flow from a tube with straight fins of rectangular profile appears to be of a significant magnitude when compared with that from a plain tube of the same outside diameter and length; a six-fold increase in heat transfer coefficient seems possible for the cases dealt with if the heat transfer coefficient is based on the plain tube outer surface area and the wall superheat over this area.

PIN FINS

A cylindrical pin fin of a constant diameter d as shown in Fig. 1c is now to be considered. If [hd/(2k)], the Biot number for the fin, is less than 0.1, the effect on the rate of heat flow from the fin of heat conduction in the radial direction is of the second order of importance (i.e. less than 1%) [17]. It is assumed that there is one-dimensional steady-state heat conduction; a uniform thermal conductivity for fin material; that there are no heat sources in the fin itself; and that the heat losses from the fin tip are negligible. If saturated nucleate pool boiling takes place on the fin, the differential equation of the temperature distribution in it is given by equation (1), provided that u in this equation is replaced by A/C. The boundary conditions given by equations (2) and (3) hold good (see Fig. 1c) and n in



FIG. 3. Ratio of the rate of heat flow from a finned tube to that from a plain tube of the same O.D. and length.

equation (4) is equal to 2. It is a straightforward matter to show that equations (5)–(14) apply to a pin fin if u in equations (7), (11) and (14) is replaced by A/C. A in equation (13) is the cross-sectional area of the pin fin.

After introducing this modification, equations (5)–(14) apply to a regularly or an irregularly shaped fin (e.g. a prismatic bar or a rectangular parallelepiped), providing that A and C are constant along its axis.

CONCLUDING COMMENTS

During saturated nucleate pool boiling, the rate of heat flow from a tube with straight fins of rectangular profile appears to be of a significant magnitude when compared with that from a plain tube of the same O.D. and length; a six-fold increase in heat transfer coefficient seems possible for a few of the numerical examples presented if the heat transfer coefficient is based on the plain tube outer surface area and the wall superheat on this area. The finned-tube can be manufactured cheaply by extrusion. However, the results obtained by this work should be verified by experiment.

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